

অষ্টম শ্রেণির গণিত অনুশীলনী ৪.৩ – উৎপাদকে বিশ্লেষণ

1. $a^3 + 8$

সমাধান:

$$a^3 + 8$$

$$= a^3 + 2^3$$

$$= (a + 2)(a^2 - a \cdot 2 + 2^2) \quad [\text{সূত্র } a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$$

$$= (a + 2)(a^2 - 2a + 4)$$

2. $8x^3 + 343$

সমাধান:

$$8x^3 + 343$$

$$= (2x)^3 + 7^3$$

$$= (2x + 7)\{(2x)^2 - 2x \cdot 7 + 7^2\} \quad [\text{সূত্র } a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$$

$$= (2x + 7)(4x^2 - 14x + 49)$$

3. $8a^4 + 27ab^3$

সমাধান:

$$8a^4 + 27ab^3$$

$$= a(8a^3 + 27b^3) \quad [\text{কমন}]$$

$$= a\{(2a)^3 + (3b)^3\}$$

$$= a(2a + 3b)\{(2a)^2 - 2a \cdot 3b + (3b)^2\} \quad [\text{সূত্র } a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$$

$$= a(2a + 3b)(4a^2 - 6ab + 9b^2)$$

4. $8x^3 + 1$

সমাধান:

$$8x^3 + 1$$

$$= (2x)^3 + 1^3$$

$$= (2x + 1)\{(2x)^2 - 2x \cdot 1 + 1^2\} \quad [\text{সূত্র } a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$$

$$= (2x + 1)(4x^2 - 2x + 1)$$

$$5. 64a^3 - 125b^3$$

সমাধান:

$$64a^3 - 125b^3$$

$$= (4a)^3 - (5b)^3$$

$$= (4a - 5b)\{(4a)^2 + 4a \cdot 5b + (5b)^2\} \quad [\text{সূত্র } a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$$

$$= (4a - 5b)(16a^2 + 20ab + 25b^2)$$

$$6. 729a^3 - 64b^3c^6$$

সমাধান:

$$729a^3 - 64b^3c^6$$

$$= (9a)^3 - (4bc^2)^3$$

$$= (9a - 4bc^2)\{(9a)^2 + 9a \cdot 4bc^2 + (4bc^2)^2\} \quad [\text{সূত্র } a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$$

$$= (9a - 4bc^2)(81a^2 + 36abc^2 + 16b^2c^4)$$

$$7. 27a^3b^3 + 64b^3c^3$$

সমাধান:

$$27a^3b^3 + 64b^3c^3$$

$$= b^3(27a^3 + 64c^3) \quad [\text{কমন}]$$

$$= b^3\{(3a)^3 + (4c)^3\}$$

$$= b^3(3a + 4c)\{(3a)^2 - 3a \cdot 4c + (4c)^2\} \quad [\text{সূত্র } a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$$

$$= b^3(3a + 4c)(9a^2 - 12ac + 16c^2)$$

$$8. 56x^3 - 189y^3$$

সমাধান:

$$56x^3 - 189y^3$$

$$= 7(8x^3 - 27y^3) \quad [\text{কমন}]$$

$$= 7\{(2x)^3 - (3y)^3\}$$

$$= 7(2x - 3y)\{(2x)^2 + 2x \cdot 3y + (3y)^2\} \quad [\text{সূত্র } a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$$

$$= 7(2x - 3y)(4x^2 + 6xy + 9y^2)$$

$$9. 3x - 75x^3$$

সমাধান:

$$3x - 75x^3$$

$$= 3x(1 - 25x^2) \quad [\text{কমন}]$$

$$= 3x\{1^2 - (5x)^2\}$$

$$= 3x(1 + 5x)(1 - 5x) \quad [\text{সূত্র } a^2 - b^2 = (a + b)(a - b)]$$

$$10. 4x^2 - y^2$$

সমাধান:

$$4x^2 - y^2$$

$$= (2x)^2 - y^2$$

$$= (2x + y)(2x - y) \quad [\text{সূত্র } a^2 - b^2 = (a + b)(a - b)]$$

$$11. 3ay^2 - 48a$$

সমাধান:

$$3ay^2 - 48a$$

$$= 3a(y^2 - 16) \quad [\text{কমন}]$$

$$= 3a(y^2 - 4^2)$$

$$= 3a(y + 4)(y - 4) \quad [\text{সূত্র } a^2 - b^2 = (a + b)(a - b)]$$

$$12. a^2 - 2ab + b^2 - p^2$$

সমাধান:

$$a^2 - 2ab + b^2 - p^2$$

$$= (a - b)^2 - p^2 \quad [\text{সূত্র } (a - b)^2 = a^2 - 2ab + b^2]$$

$$= (a - b + p)(a - b - p) \quad [\text{সূত্র } a^2 - b^2 = (a + b)(a - b)]$$

$$13. 16y^2 - a^2 - 6a - 9$$

সমাধান:

$$\begin{aligned}
& 16y^2 - a^2 - 6a - 9 \\
& = 16y^2 - (a^2 + 6a + 9) \\
& = 16y^2 - (a^2 + 2 \cdot a \cdot 3 + 3^2) \\
& = (4y)^2 - (a + 3)^2 \quad [\text{সূত্র } (a + b)^2 = a^2 + 2ab + b^2] \\
& = \{4y + (a + 3)\}\{4y - (a + 3)\} \quad [\text{সূত্র } a^2 - b^2 = (a + b)(a - b)] \\
& = (4y + a + 3)(4y - a - 3)
\end{aligned}$$

14. $8a + ap^3$

সমাধান:

$$\begin{aligned}
& 8a + ap^3 \\
& = a(8 + p^3) \quad [\text{কমন}] \\
& = a(p^3 + 8) \\
& = a\{p^3 + 2^3\} \\
& = a(p + 2)(p^2 - p \cdot 2 + 2^2) \quad [\text{সূত্র } a^3 + b^3 = (a + b)(a^2 - ab + b^2)] \\
& = a(p + 2)(p^2 - 2p + 4)
\end{aligned}$$

15. $2a^3 + 16b^3$

সমাধান:

$$\begin{aligned}
& 2a^3 + 16b^3 \\
& = 2(a^3 + 8b^3) \quad [\text{কমন}] \\
& = 2\{a^3 + (2b)^3\} \\
& = 2(a + 2b)\{a^2 - a \cdot 2b + (2b)^2\} \quad [\text{সূত্র } a^3 + b^3 = (a + b)(a^2 - ab + b^2)] \\
& = 2(a + 2b)(a^2 - 2ab + 4b^2)
\end{aligned}$$

16. $x^2 + y^2 - 2xy - 1$

সমাধান:

$$\begin{aligned}
& x^2 + y^2 - 2xy - 1 \\
& = x^2 - 2xy + y^2 - 1 \\
& = (x - y)^2 - 1^2 \quad [\text{সূত্র } (a - b)^2 = a^2 - 2ab + b^2]
\end{aligned}$$

$$= (x - y + 1)(x - y - 1)$$

$$[\text{সূত্র } a^2 - b^2 = (a + b)(a - b)]$$

$$17. a^2 - 2ab + 2b - 1$$

সমাধান:

$$a^2 - 2ab + 2b - 1$$

$$= a^2 - 1 - 2ab + 2b$$

$$= a^2 - 1^2 - 2b(a - 1) \quad [\text{কমন}]$$

$$= (a + 1)(a - 1) - 2b(a - 1) \quad [\text{সূত্র } a^2 - b^2 = (a + b)(a - b)]$$

$$= (a - 1)(a + 1 - 2b) \quad [\text{কমন}]$$

$$= (a - 1)(a - 2b + 1)$$

$$18. x^4 - 2x^2 + 1$$

সমাধান:

$$x^4 - 2x^2 + 1$$

$$= (x^2)^2 - 2 \cdot x^2 \cdot 1 + 1^2$$

$$= (x^2 - 1)^2 \quad [\text{সূত্র } (a - b)^2 = a^2 - 2ab + b^2]$$

$$= (x^2 - 1^2)^2$$

$$= \{(x + 1)(x - 1)\}^2 \quad [\text{সূত্র } a^2 - b^2 = (a + b)(a - b)]$$

$$= (x + 1)^2(x - 1)^2$$

$$19. 36 - 12x + x^2$$

সমাধান:

$$36 - 12x + x^2$$

$$= x^2 - 12x + 36$$

$$= x^2 - 2 \cdot x \cdot 6 + 6^2$$

$$= (x - 6)^2 \quad [\text{সূত্র } (a - b)^2 = a^2 - 2ab + b^2]$$

$$20. x^6 - y^6$$

সমাধান:

$$x^6 - y^6$$

$$= (x^3)^2 - (y^3)^2$$

$$= (x^3 + y^3)(x^3 - y^3) \quad [\text{সূত্র } a^2 - b^2 = (a + b)(a - b)]$$

$$= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$$

$$[\text{সূত্র } a^3 + b^3 = (a + b)(a^2 - ab + b^2)] \text{ এবং } [\text{সূত্র } a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$$

$$= (x + y)(x - y)(x^2 - xy + y^2)(x^2 + xy + y^2)$$

$$21. (x - y)^3 + z^3$$

সমাধান:

$$(x - y)^3 + z^3$$

$$= \{(x - y) + z\}\{(x - y)^2 - z(x - y) + z^2\} \quad [\text{সূত্র } a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$$

$$= (x - y + z)(x^2 - 2xy + y^2 - xz + yz + z^2) \quad [\text{সূত্র } (a - b)^2 = a^2 - 2ab + b^2]$$

$$= (x - y + z)(x^2 - 2xy - xz + y^2 + yz + z^2)$$

$$22. 64x^3 - 8y^3$$

সমাধান:

$$64x^3 - 8y^3$$

$$= 8(8x^3 - y^3) \quad [\text{কমন}]$$

$$= 8\{(2x)^3 - y^3\}$$

$$= 8(2x - y)\{(2x)^2 + 2x \cdot y + y^2\} \quad [\text{সূত্র } a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$$

$$= 8(2x - y)(4x^2 + 2xy + y^2)$$

$$23. x^2 + 14x + 40$$

সমাধান:

$$x^2 + 14x + 40$$

$$= x^2 + 10x + 4x + 40 \quad [\text{মিডল টার্ম ব্রেক}]$$

$$= x(x + 10) + 4(x + 10) \quad [\text{কমন}]$$

$$= (x + 10)(x + 4) \quad [\text{কমন}]$$

$$24. x^2 + 7x - 120$$

সমাধান:

$$x^2 + 7x - 120$$

$$= x^2 + 15x - 8x - 120 \quad [\text{মিডল টার্ম ব্রেক}]$$

$$= x(x + 15) - 8(x + 15) \quad [\text{কমন}]$$

$$= (x + 15)(x - 8) \quad [\text{কমন}]$$

$$25. x^2 - 51x + 650$$

সমাধান:

$$x^2 - 51x + 650$$

$$= x^2 - 26x - 25x + 650 \quad [\text{মিডল টার্ম ব্রেক}]$$

$$= x(x - 26) - 25(x - 26) \quad [\text{কমন}]$$

$$= (x - 26)(x - 25) \quad [\text{কমন}]$$

$$26. a^2 + 7ab + 12b^2$$

সমাধান:

$$a^2 + 7ab + 12b^2$$

$$= a^2 + 4ab + 3ab + 12b^2 \quad [\text{মিডল টার্ম ব্রেক}]$$

$$= a(a + 4b) + 3b(a + 4b) \quad [\text{কমন}]$$

$$= (a + 4b)(a + 3b) \quad [\text{কমন}]$$

$$27. p^2 + 2pq - 80q^2$$

সমাধান:

$$p^2 + 2pq - 80q^2$$

$$= p^2 + 10pq - 8pq - 80q^2 \quad [\text{মিডল টার্ম ব্রেক}]$$

$$= p(p + 10q) - 8q(p + 10q) \quad [\text{কমন}]$$

$$= (p + 10q)(p - 8q) \quad [\text{কমন}]$$

$$28. x^2 - 3xy - 40y^2$$

সমাধান:

$$x^2 - 3xy - 40y^2$$

$$= x^2 - 8xy + 5xy - 40y^2 \quad [\text{মিডল টার্ম ব্রেক}]$$

$$= x(x - 8y) + 5y(x - 8y) \quad [\text{কমন}]$$

$$= (x - 8y)(x + 5y) \quad [\text{কমন}]$$

$$29. (x^2 - x)^2 + 3(x^2 - x) - 40$$

সমাধান:

$$\text{ধরি, } t = x^2 - x$$

$$\text{তাহলে, } t^2 + 3t - 40$$

$$= t^2 + 8t - 5t - 40 \quad [\text{মিডল টার্ম ব্রেক}]$$

$$= t(t + 8) - 5(t + 8) \quad [\text{কমন}]$$

$$= (t + 8)(t - 5) \quad [\text{কমন}]$$

$$= (x^2 - x + 8)(x^2 - x - 5) \quad [t = x^2 - x \text{ বসিয়ে}]$$

$$30. (a^2 + b^2)^2 - 18(a^2 + b^2) - 88$$

সমাধান:

$$\text{ধরি, } t = a^2 + b^2$$

$$\text{তাহলে, } t^2 - 18t - 88$$

$$= t^2 - 22t + 4t - 88 \quad [\text{মিডল টার্ম ব্রেক}]$$

$$= t(t - 22) + 4(t - 22) \quad [\text{কমন}]$$

$$= (t - 22)(t + 4) \quad [\text{কমন}]$$

$$= (a^2 + b^2 - 22)(a^2 + b^2 + 4) \quad [t = a^2 + b^2 \text{ বসিয়ে}]$$

$$31. (a^2 + 7a)^2 - 8(a^2 + 7a) - 180$$

সমাধান:

$$\text{ধরি, } t = a^2 + 7a$$

$$\text{তাহলে, } t^2 - 8t - 180$$

$$= t^2 - 18t + 10t - 180 \quad [\text{মিডল টার্ম ব্রেক}]$$

$$\begin{aligned}
&= t(t - 18) + 10(t - 18) \quad [\text{কমন}] \\
&= (t - 18)(t + 10) \quad [\text{কমন}] \\
&= (a^2 + 7a - 18)(a^2 + 7a + 10) \quad [t = a^2 + 7a \text{ বসিয়ে}] \\
&= (a^2 + 9a - 2a - 18)(a^2 + 5a + 2a + 10) \quad [\text{মিডল টার্ম ব্রেক}] \\
&= \{a(a + 9) - 2(a + 9)\}\{a(a + 5) + 2(a + 5)\} \quad [\text{কমন}] \\
&= (a + 9)(a - 2)(a + 5)(a + 2) \quad [\text{কমন}] \\
&= (a + 2)(a - 2)(a + 9)(a + 5) \\
\mathbf{32.} & x^2 + (3a + 4b)x + (2a^2 + 5ab + 3b^2)
\end{aligned}$$

সমাধান:

$$\begin{aligned}
&x^2 + (3a + 4b)x + (2a^2 + 5ab + 3b^2) \\
&= x^2 + (3a + 4b)x + (2a^2 + 3ab + 2ab + 3b^2) \quad [\text{মিডল টার্ম ব্রেক}] \\
&= x^2 + (3a + 4b)x + \{a(2a + 3b) + b(2a + 3b)\} \quad [\text{কমন}] \\
&= x^2 + (3a + 4b)x + (2a + 3b)(a + b) \quad [\text{কমন}] \\
&= x^2 + (2a + 3b)x + (a + b)x + (a + b)(2a + 3b) \quad [\text{মিডল টার্ম ব্রেক}] \\
&= x(x + 2a + 3b) + (a + b)(x + 2a + 3b) \quad [\text{কমন}] \\
&= (x + 2a + 3b)(x + a + b) \quad [\text{কমন}]
\end{aligned}$$

$$\mathbf{33.} \quad 6x^2 - x - 15$$

সমাধান:

$$\begin{aligned}
&6x^2 - x - 15 \\
&= 6x^2 - 10x + 9x - 15 \quad [\text{মিডল টার্ম ব্রেক}] \\
&= 2x(3x - 5) + 3(3x - 5) \quad [\text{কমন}] \\
&= (3x - 5)(2x + 3) \quad [\text{কমন}]
\end{aligned}$$

$$\mathbf{34.} \quad x^2 - x - (a + 1)(a + 2)$$

সমাধান:

$$x^2 - x - (a + 1)(a + 2)$$

$$= x^2 - (a+2)x + (a+1)x - (a+1)(a+2) \quad [\text{মিডল টার্ম ব্রেক}]$$

$$= x\{x - (a+2)\} + (a+1)\{x - (a+2)\} \quad [\text{কমন}]$$

$$= \{x - (a+2)\}\{x + (a+1)\} \quad [\text{কমন}]$$

$$= (x - a - 2)(x + a + 1)$$

$$35. 3x^2 + 11x - 4$$

সমাধান:

$$3x^2 + 11x - 4$$

$$= 3x^2 + 12x - x - 4 \quad [\text{মিডল টার্ম ব্রেক}]$$

$$= 3x(x+4) - 1(x+4) \quad [\text{কমন}]$$

$$= (x+4)(3x-1) \quad [\text{কমন}]$$

$$36. 3x^2 - 16x - 12$$

সমাধান:

$$3x^2 - 16x - 12$$

$$= 3x^2 - 18x + 2x - 12 \quad [\text{মিডল টার্ম ব্রেক}]$$

$$= 3x(x-6) + 2(x-6) \quad [\text{কমন}]$$

$$= (x-6)(3x+2) \quad [\text{কমন}]$$

$$37. 2x^2 - 9x - 35$$

সমাধান:

$$2x^2 - 9x - 35$$

$$= 2x^2 - 14x + 5x - 35 \quad [\text{মিডল টার্ম ব্রেক}]$$

$$= 2x(x-7) + 5(x-7) \quad [\text{কমন}]$$

$$= (x-7)(2x+5) \quad [\text{কমন}]$$

$$38. 2x^2 - 5xy + 2y^2$$

সমাধান:

$$2x^2 - 5xy + 2y^2$$

$$= 2x^2 - 4xy - xy + 2y^2 \quad [\text{মিডল টার্ম ব্রেক}]$$

$$= 2x(x - 2y) - y(x - 2y) \quad [\text{কমন}]$$

$$= (x - 2y)(2x - y) \quad [\text{কমন}]$$

$$39. x^3 - 8(x - y)^3$$

সমাধান:

$$x^3 - 8(x - y)^3$$

$$= x^3 - \{2(x - y)\}^3$$

$$= \{x - 2(x - y)\}[x^2 + x \cdot 2(x - y) + \{2(x - y)\}^2] \quad [\text{সূত্র } a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$$

$$= (x - 2x + 2y)\{x^2 + 2x(x - y) + 4(x - y)^2\}$$

$$= (-x + 2y)\{x^2 + 2x^2 - 2xy + 4(x^2 - 2xy + y^2)\} \quad [\text{সূত্র } (a - b)^2 = a^2 - 2ab + b^2]$$

$$= (2y - x)(x^2 + 2x^2 - 2xy + 4x^2 - 8xy + 4y^2)$$

$$= (2y - x)(7x^2 - 10xy + 4y^2)$$

$$40. 10p^2 + 11pq - 6q^2$$

সমাধান:

$$10p^2 + 11pq - 6q^2$$

$$= 10p^2 + 15pq - 4pq - 6q^2 \quad [\text{মিডল টার্ম ব্রেক}]$$

$$= 5p(2p + 3q) - 2q(2p + 3q) \quad [\text{কমন}]$$

$$= (2p + 3q)(5p - 2q) \quad [\text{কমন}]$$

$$41. 2(x + y)^2 - 3(x + y) - 2$$

সমাধান:

$$\text{ধরি, } t = x + y$$

$$\text{তাহলে, } 2t^2 - 3t - 2$$

$$= 2t^2 - 4t + t - 2 \quad [\text{মিডল টার্ম ব্রেক}]$$

$$= 2t(t - 2) + (t - 2) \quad [\text{কমন}]$$

$$= (2t + 1)(t - 2) \quad [\text{কমন}]$$

$$= \{2(x + y) + 1\}(x + y - 2) \quad [t = x + y \text{ বসিয়ে}]$$

$$= (2x + 2y + 1)(x + y - 2)$$

$$42. ax^2 + (a^2 + 1)x + a$$

সমাধান:

$$ax^2 + (a^2 + 1)x + a$$

$$= ax^2 + a^2x + x + a$$

$$= ax(x + a) + 1(x + a) \quad [\text{কমন}]$$

$$= (x + a)(ax + 1) \quad [\text{কমন}]$$

$$43. 15x^2 - 11xy - 12y^2$$

সমাধান:

$$15x^2 - 11xy - 12y^2$$

$$= 15x^2 - 20xy + 9xy - 12y^2 \quad [\text{মিডল টার্ম ব্রেক}]$$

$$= 5x(3x - 4y) + 3y(3x - 4y) \quad [\text{কমন}]$$

$$= (3x - 4y)(5x + 3y) \quad [\text{কমন}]$$

$$44. a^3 - 3a^2b + 3ab^2 - 2b^3$$

সমাধান:

$$a^3 - 3a^2b + 3ab^2 - 2b^3$$

$$= a^3 - 3a^2b + 3ab^2 - b^3 - b^3$$

$$= (a - b)^3 - b^3 \quad [\text{সূত্র } (a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3]$$

$$= (a - b - b)\{(a - b)^2 + (a - b).b + b^2\}$$

$$= (a - 2b)(a^2 - 2ab + b^2 + ab - b^2 + b^2) \quad [\text{সূত্র } (a - b)^2 = a^2 - 2ab + b^2]$$

$$= (a - 2b)(a^2 - ab + b^2)$$